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Analysis of decay data from neutron-rich nuclei

U.C. Bergmann[1], M.J.G. Borge[2], J. Cederkäll[3], C. Forssén[1], E. Fumero[4], H.O.U. Fynbo[3], H. Gausemel[5], H. Jeppesen[6], B. Jonson[1], K. Markenroth[1], T. Nilsson[3], G. Nyman[1], K. Riisager[6,a], H. Simon[7], O. Tengblad[2], L. Weissman[3], F. Wenander[1], K. Wilhelmsen Rolander[4], and the ISOLDE Collaboration[3]

[1] Experimentell Fysik, Chalmers Tekniska Högskola, S-412 96 Göteborg, Sweden
[2] Instituto Estructura de la Materia, CSIC, E-28006 Madrid, Spain
[3] EP Division, CERN, CH-1211 Genève 23, Switzerland
[4] Fysiska Institutionen, Stockholms Universitet, Box 6730, S-113 85 Stockholm, Sweden
[5] Kjemisk Institutt, Universitetet i Oslo, Postboks 1033, Blindern, N-0315 Oslo, Norway
[6] Institut for Fysik og Astronomi, Aarhus Universitet, DK-8000 Aarhus C, Denmark
[7] Institut für Kernphysik, Technische Universität, D-64289 Darmstadt, Germany

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Abstract. The β-decays of the neutron-rich nuclei ^{12}Be and ^{29}Ne have been studied. The statistical correlations between the almost identical half-lives of ^{12}Be and its daughter ^{12}B are analysed for a large sample of ^{12}Be decay data. Stringent mutual bounds are obtained on the parameter set, leading to a precise determination of the ^{12}Be half-life of 21.50 ± 0.04 ms. From a simultaneous detection of β-particles and neutrons from the decay of ^{29}Ne the neutron emission probability, P_n, is determined to $17 \pm 5\%$. No indication of two-neutron emission is seen from this nucleus. An upper limit of 2.2% (90% confidence level) is established for P_{2n}.

PACS. 23.40.-s Beta decay; double beta decay; electron and muon capture – 27.20.+n $6 \leq A \leq 19$ – 27.30.+t $20 \leq A \leq 38$ – 07.05.Kf Data analysis: algorithms and implementation; data management

1 Introduction

Much effort is presently invested in increasing our knowledge on very neutron-rich light nuclei close to the drip-line. Both in-flight and ISOL facilities contribute in a complementary manner. Basic decay characteristics are among the first properties to be measured and these can provide insight on nuclear structure if measured accurately and reliably. The present paper reports on two nuclei where increased accuracy has been obtained and is part of our systematic investigation of the β strength distribution for nuclei close to the neutron drip-line. New aspects of nuclear structure may be revealed here, see *e.g.* [1–3].

When aiming for higher accuracy, one must optimise also the statistical treatment of the data [4]. We present here two different examples of this. In sect. 2, on the half-life of ^{12}Be, the data have a reasonably high statistics, but the fact that the daughter has an almost identical half-life to its parent drastically reduces the precision that can be extracted from a single experiment. This is an extreme case of a situation that is encountered in several chain decays and we expose the correlations between the two half-lives explicitly. The second case, discussed in sect. 3, concerns the β-delayed neutron branches of ^{29}Ne. Here the statistics is limited and possible background contributions have to be estimated carefully. In both cases the precision of decay parameters for the daughter nuclei turn out to be a limiting factor.

The experimental setup used for both nuclei is described in [3] where the method of analysis for ^{29}Ne can also be found.

[a] e-mail: kvr@ifa.au.dk

2 Half-life of ^{12}Be

During an experiment at ISOLDE/CERN a total amount of 5×10^7 ^{12}Be ions were collected from a radioactive beam generated by a pulsed 1 GeV beam of 3×10^{13} protons per pulse on a uranium carbide target. With a plastic scintillator and a neutron long counter, using "singles" time multi-scaling, the time structures of β-particles and β-delayed neutrons were repeatedly registered relative to proton beam impact and accumulated for many proton pulses, see [3] for details. Since the experiment was aiming at a precision level of 1 per mille, an absolute calibration of the intrinsic time scale was required. This was obtained by using a macroscopic time signal, generated

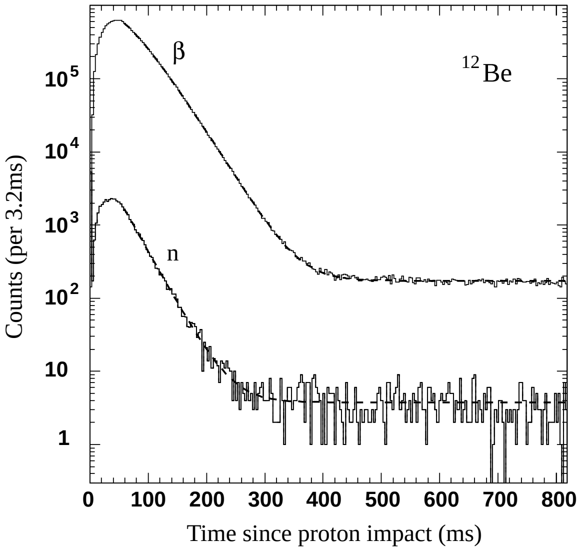

Fig. 1. Measured time distributions, relative to proton beam impact, of β-particles and β-delayed neutrons from the decay of ^{12}Be. The ^{12}Be ion beam was shut off at times $t > 53$ ms to obtain pure decay components. The dashed curves show the results of fits to the histograms with bin size 0.1 ms (it is 3.2 ms in the figure). The β spectrum was treated as a chain decay of ^{12}Be through ^{12}B, while only ^{12}Be gives neutrons.

every 1.2 s by the CERN Proton Synchrotron Booster to mark the impact of a proton pulse, to reset the intrinsic clock. In this way a beam of a long-lived radioactivity allowed the apparatus time scale to be verified to a precision of 5×10^{-4}.

The data from the ^{12}Be measurement are shown in fig. 1. Each of the two histograms is a combination of seven individually accumulated decay spectra. An electrostatic deflector was used to steer away the ion beam in order to optimise the collection for the half-life measurement. This "beam gate" was repeatedly closed at $t_0 = 53$ ms after proton impact, *i.e.* when the neutron activity was near maximum, implying a maximum sample size. Time spectra for both neutrons and β-particles were fitted with theoretical expressions in the region $t > t_0$ using the "Poisson likelihood χ^2" statistic, denoted χ^2_λ, which combines the principle of maximum likelihood for estimation and the likelihood ratio for goodness-of-fit testing [5]. For the multi-parameter minimisation of χ^2_λ and the analysis of parameter errors and correlations the code MINUIT [6] was used.

The neutron spectrum was fitted with an exponential component plus a constant background, giving a half-life of 21.18 ± 0.17 ms for ^{12}Be. The β spectrum was analysed as a chain decay of ^{12}Be through its daughter ^{12}B assuming initial amounts of both nuclei, $N_1(t_0) = N_1$ and $N_2(t_0) = N_2$, respectively. The amounts of parent and daughter nuclei in the sample at time t after each proton pulse are governed by the coupled equations

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = -\lambda_1 N_1(t)\,, \tag{1}$$

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = -\lambda_2 N_2(t) + (1 - P_n)\lambda_1 N_1(t)\,, \tag{2}$$

where λ_1 and λ_2 are the relevant decay constants and P_n is the neutron branch from ^{12}Be. Solving the differential equations for $t > t_0$, yields

$$N_1(t) = N_1 e^{-\lambda_1 (t-t_0)}\,, \tag{3}$$

$$N_2(t) = \left(N_2 + \frac{N_1\lambda_1}{\lambda_1 - \lambda_2}(1 - P_n)\right) e^{-\lambda_2(t-t_0)} - \frac{N_1\lambda_1}{\lambda_1 - \lambda_2}(1 - P_n) e^{-\lambda_1(t-t_0)}\,. \tag{4}$$

The β activity can be expressed as [1]

$$R_\beta(t) = \varepsilon_\beta \lambda_1 N_1(t) + \varepsilon_\beta \lambda_2 N_2(t) + R_\beta^{\mathrm{back}}\,. \tag{5}$$

The maximum detection rates for β-particles and neutrons (at time t_0) were $0.06/\mu$s and $0.0003/\mu$s, respectively. The dead time in the electronics and data acquisition is of the order of 1 μs and, apparently, it does not influence the neutron measurement. In order to describe the effects of dead time on the measured time characteristics for β-particles, we need to consider changes in the β activity caused by (non-statistical) variations of the ^{12}Be yield during data taking. Such variations will not alter the time shape of the activity, as described by eq. (5), but only its amplitude, *i.e.* $R_\beta(t) = \alpha\, p(t)$ where α may vary from proton pulse to proton pulse but the function $p(t)$ remains unchanged. This is due to the fact that the time distribution of ^{12}Be ions extracted from the target and ion source follows, to a great extent, a universal release function [3], which makes N_2 universally proportional to N_1.[2] Thus, the count numbers in the β spectrum of fig. 1 can be described by theoretical values of

$$N_{\mathrm{bin}} = \int_{\mathrm{bin}} \mathrm{d}t N_{\mathrm{pulse}} R_\beta(t) \left(1 - \tau_{\mathrm{eff}} R_\beta(t)\right)\,, \tag{6}$$

where

$$\tau_{\mathrm{eff}} = \tau \left(1 + \frac{\mathrm{Var}(\alpha)}{\langle\alpha\rangle^2}\right)\,. \tag{7}$$

Here, N_{pulse} is the total number of proton pulses. The effective dead-time parameter, τ_{eff}, is larger than the true dead time, τ, by an amount depending on the variance of α relative to its mean square. Major fluctuations in α happened only when the laser ionisation was optimised, *i.e.* in between the accumulation of two decay spectra.

[1] The efficiency of the β counter, ε_β, can be determined from the ratio of β-particles to neutrons but is not important in this context.

[2] The influence of the background component has been neglected.

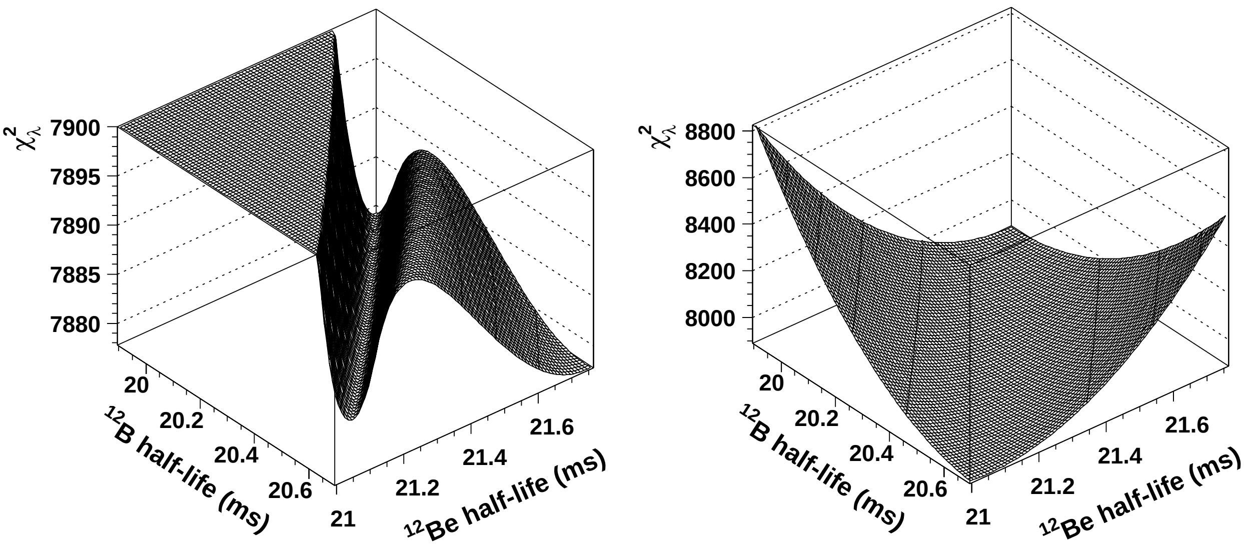

Fig. 2. For the β-decay data the surface of "Poisson likelihood χ^2" (χ^2_λ) [5] is shown in the two-dimensional parameter space of ^{12}B and ^{12}Be half-lives, left: for the case where χ^2_λ was minimised with respect to the dead-time parameter τ_{eff} (zoomed on the region of $\chi^2_\lambda < 7900$), and right: for a fixed value of τ_{eff}. In both cases χ^2_λ was minimised with respect to the remaining parameters of eqs. (3)-(6). The half-life parameters appear to be strongly correlated and the physical half-lives should be found at the bottom of the valley, visible in the central part of both panels. Further explanation is given in the text.

Therefore, the lower limit obtained by averaging over the seven spectra, $1 + \text{Var}(\alpha)/\langle\alpha\rangle^2 = 1.19$, gives a reliable estimate of τ_{eff}/τ.

Note that we introduce a small error by still assuming a Poisson distribution for the dead-time influenced data. We believe that this error in our case can be neglected since the final half-life results are essentially unaffected by the dead-time correction. A more detailed discussion can be found in [7] where the alternative procedure of fitting each spectrum separately is tested. However, there a Gaussian weighting is used which (as also recognized in the paper) introduces biases [8]. When aiming for very high accuracy it would be wise to follow ref. [7] and employ Monte Carlo simulations of the data and the analysis procedure.

The β spectrum was fitted with the theoretical values of eq. (6), the expressions of eqs. (3)-(5) substituted, by varying the seven parameters N_1, N_2, $t_{1/2,1}$, $t_{1/2,2}$, P_n, R_β^{back}, and τ_{eff}, where $t_{1/2,i} = \ln 2/\lambda_i$. Variations of the P_n value, within the literature confidence interval $0.50 \pm 0.03\%$ [3], had no influence on the fit. The left panel of fig. 2 shows the χ^2_λ function in the $(t_{1/2,2}, t_{1/2,1})$ parameter space. At each point $(t_{1/2,2}, t_{1/2,1})$ the χ^2_λ function was minimised with respect to all remaining parameters. The contours of constant χ^2_λ are highly eccentric ellipses oriented at approximately 45° angle to the half-life axes. This means that the ^{12}Be and ^{12}B half-lives are strongly correlated. In fact, the global correlation coefficients for N_1, N_2, $t_{1/2,1}$, $t_{1/2,2}$, and τ_{eff} are all greater than 0.998. This indicates both an exceptionally difficult problem, and one which has been badly parametrised so that individual errors are not very meaningful because they are so highly correlated [6,9]. The region of low χ^2_λ at high values of both half-lives in fig. 2 (left panel) is non-physical and is caused by the introduction of the freely varying dead-time parameter τ_{eff} which here takes on unrealistic values (higher than 5 μs). On the other hand, the valley of low χ^2_λ (oriented in the direction of the $t_{1/2,2} = -t_{1/2,1}$ line) is the region of physical half-lives where τ_{eff} is of the order of 1 μs. This is also indicated by the right panel of fig. 2 which shows the χ^2_λ function for a fixed value of τ_{eff}.

The steep rise of χ^2_λ, along the $t_{1/2,2} = t_{1/2,1}$ line on both sides of the valley in fig. 2, reflects the high quality of the data. It shows that along this direction the half-lives are simultaneously very well determined. For instance, the central value of the half-lives given in the literature, (20.20 ms, 21.30 ms) [10], can be excluded with high confidence. However, if we instead choose the 2σ upper limit on the literature ^{12}Be half-life, 21.5 ms, we enter the valley of low confidence. Thus, the present β-decay data are not in strict contradiction with the literature, and neither with the present neutron data, but add a substantial amount of information if one knows either of the two half-lives with high certainty.

According to the literature, the ^{12}B half-life is known with high accuracy, 20.20 ± 0.02 ms. This value is based on a non-linear least-squares fit of an exponential plus constant to a large sample of β-decay data [11]. It is well-known that least-squares minimisation with "Pearson's χ^2" or "Neyman's χ^2" statistics (their difference being the use of either theoretical or experimental errors) introduces a bias on the parameter estimate which, however, disappears in the asymptotic limit of large samples [8]. To check whether this could have influenced the estimate of [11], a series of time spectra, with the same number of counts in the exponential and constant background as obtained by [11], were simulated using Monte Carlo techniques. From a χ^2 analysis of these spectra it was con-

Table 1. Half-lives of ^{12}B and ^{12}Be from the literature [10] and from a fit of the measured β spectrum (fig. 1) using the theoretical expression of eq. (6), combined with eqs. (3)-(5). During the minimisation of χ^2_λ, the ^{12}B half-life was fixed at the literature value whereas all other parameters were varied freely.

	Literature	Beta-decay data
^{12}B half-life (ms)	20.20 ± 0.02	20.20 FIXED
^{12}Be half-life (ms)	21.30 ± 0.10	21.502 ± 0.028

cluded that, in the worst case, a bias of +0.010 ms has been introduced, which is negligible.

Since earlier experiments have established the ^{12}B half-life with higher precision than the ^{12}Be half-life, we adopt the former in order to give an improved estimate of the latter. When fixing the ^{12}B half-life, the correlations of the ^{12}Be half-life with other parameters disappear (the global correlation drops to 0.89). The result of the fit is presented and compared with literature values in table 1. Taking into account the 0.02 ms uncertainty on the ^{12}B half-life, we arrive at a 1σ confidence interval of

$$t_{1/2}(^{12}\mathrm{Be}) = 21.50 \pm 0.04 \text{ ms}. \quad (8)$$

The χ^2_λ value per degrees of freedom, n, was checked separately in the decay region (below 300 ms) and for the constant background (above 520 ms). The former was 2394/2446 which agrees with a χ^2 probability distribution of χ^2_λ. The latter was 3203/2988, in perfect agreement with the rise in χ^2_λ/n expected for a uniform spectrum with Poisson mean value of 5 counts per channel [4].

At this point, there is no reason to believe that the literature ^{12}B half-life is wrong, but in case improvements are made in the future we give a parameterisation of the ^{12}Be half-life estimated (everywhere with an error of 0.028 ms) from different fixed values of the ^{12}B half-life:

$$t_{1/2}(^{12}\mathrm{Be}) = 3.2852 + 2.6777\, t_{1/2}(^{12}\mathrm{B}) - 0.087916\, t_{1/2}(^{12}\mathrm{B})^2 . \quad (9)$$

This quadratic relation, which defines the bottom of the χ^2_λ valley in fig. 2, is insensitive to changes of the dead-time parameter τ_{eff} which shows that, in the present approach, the ^{12}Be half-life is a truly uncorrelated parameter.

The half-life deduced from the neutron data is compatible with our final value deduced from the β data in eq. (8) within two standard deviations. The latter value is much more precise and we recommend to use this rather than an average of the two measurements.

3 Beta-delayed neutron branches from ^{29}Ne

The β-decay of ^{29}Ne was studied with a setup and experimental conditions similar to the ones applied in the experiment on β-delayed neutron emission from ^{12,14}Be [3] (see also the brief description in sect. 2). The isotopes of noble gases, produced by pulsed proton bombardment of a heated uranium carbide target, were transported via a cooled transfer line to a plasma discharge ion source. After extraction with a voltage of 60 kV the ^{29}Ne ion beam was mass separated with the General Purpose Separator (GPS, M/ΔM $\simeq$ 500 resolving power) during a combined measurement of P_n and P_{2n}, and with the High-Resolution Separator (HRS, M/ΔM $\simeq$ 6000) during an additional measurement of P_{2n}. The GPS and HRS beam intensities were 1.0 and 0.4 ^{29}Ne ions/s, respectively.

Figure 3 shows the distributions of detection times, relative to the impact of the preceding proton pulse, of β-particles and neutrons. The number of counts in these spectra, the background levels subtracted, are referred to as N_β and N_n, respectively. We cannot extract a reliable half-life for ^{29}Ne from our data since we collect activity until 100 ms, but our data are compatible with the literature half-lives quoted in table 2. An earlier ISOLDE measurement [14] reported a longer-lived neutron activity on mass 29, but since the separator resolution was less at that time this activity was most likely due to collection of ^{145}Xe^{5+} and decay from ^{145}Xe and its daughter ^{145}Cs.

Also shown in fig. 3 is the distribution of time separations between β-particles and subsequent neutrons. Due to the thermalisation in the long counter, the detection of neutrons is exponentially delayed with a mean residence time of 89 ± 1 μs [3]. The number of correlated βn pairs in the exponential, $N_{\beta n}$, is proportional to P_n. Similarly, one can construct the spectrum of n-n time separations and determine P_{2n} from a fit with an exponential (plus a constant) yielding the number of correlated n-n pairs, N_{nn}.

The four observables N_β, N_n, $N_{\beta n}$ and N_{nn}, which include contributions from both ^{29}Ne and the Na daughters, are connected to the various neutron branches as follows:

$$N_\beta = 2N_0\varepsilon_\beta\,, \quad (10)$$

$$N_n = N_0\varepsilon_n\left(P_n + \sum_{i=0}^{2} P_{in}P_n^{(i)}\right), \quad (11)$$

$$N_{\beta n} = N_0\varepsilon_\beta\varepsilon_n\left(P_n + \sum_{i=0}^{2} P_{in}P_n^{(i)}\right), \quad (12)$$

$$N_{nn} = N_0\varepsilon_n^2(P_{2n} + P_{0n}P_{2n}^{(0)})\,, \quad (13)$$

where N_0 is the number of parent decays. The neutron branches of the $\beta(in)$ daughters have been given indices (i). Relevant literature information on ^{29}Ne and the involved Na isotopes is collected in table 2. The only short-lived grand-daughter is ^{29}Mg ($t_{1/2} = 1.30 \pm 0.12$ s [10]) whose contribution of β-particles in the time region 0–200 ms was estimated and subtracted when calculating N_β and its uncertainty. Solving eqs. (10)-(13) with respect to P_n and P_{2n} of ^{29}Ne, using $P_{0n} = 1 - P_n + P_{2n}$

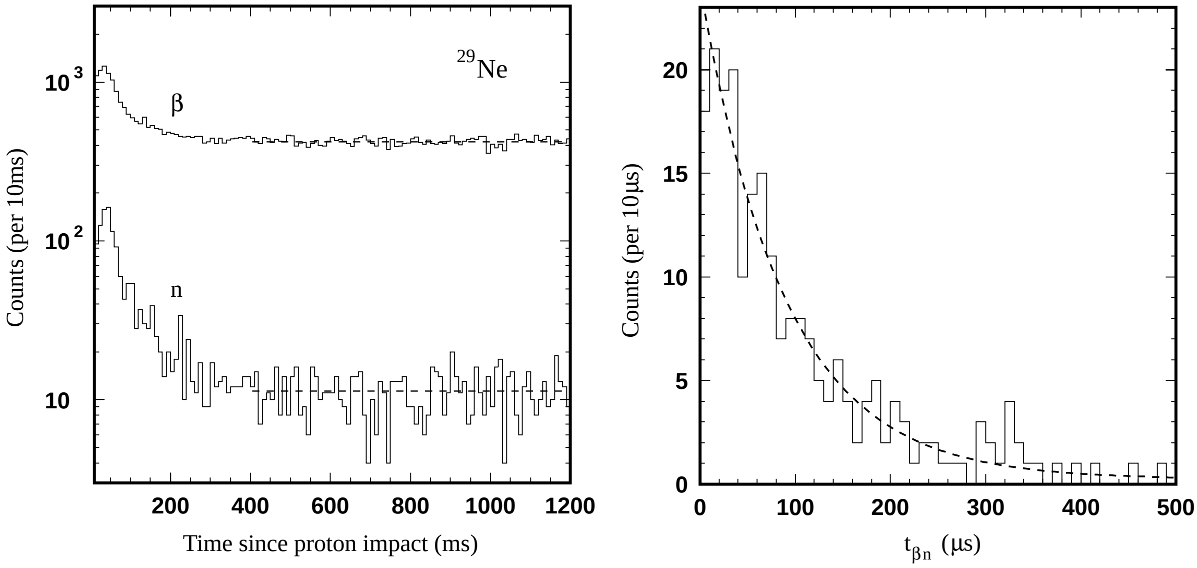

Fig. 3. Left: detection times, relative to proton beam impact, of β-particles and delayed neutrons from ^{29}Ne. The dashed lines show the fitted background levels. Right: time separations between β-particles and all subsequent neutrons arriving within 500 μs correlation time. Only particles registered within the first 200 ms after proton impact were selected. From an exponential fit, with a constant background added, the number of correlated βn pairs was determined.

Table 2. Half-lives and neutron branches relevant for the β-decay of ^{29}Ne, taken from literature or determined in this work from two independent data sets. For P_{2n}, upper limits are given at 90% confidence level.

	Literature		GPS data		HRS data
	$t_{1/2}$(ms)	P_n(%)	P_n(%)	P_{2n}(%)	P_{2n}(%)
^{29}Ne	15.6 ± 0.5^a	27 ± 9^b	17 ± 5	< 2.9 (90% C.L.)	< 2.2 (90% C.L.)
^{29}Na	44.9 ± 1.2^c	25.9 ± 2.3^c			
^{28}Na	30.5 ± 0.4^c	0.58 ± 0.12^c			

^{a} From [12]; ^{b} from [13]; ^{c} from [10].

and $P_{1n} = P_n - 2P_{2n}$, yields

$$P_n = \frac{1}{1 - P_n^{(0)} + P_n^{(1)}} \Bigg[- P_n^{(0)} + \frac{2N_{\beta n}}{\varepsilon_n N_\beta} \left(1 - \left(P_n^{(0)} - 2P_n^{(1)} \right) \frac{N_{nn}}{\varepsilon_n N_n} \right) \Bigg], \qquad (14)$$

$$P_{2n} = \frac{N_{nn}}{\varepsilon_n N_n} \left(P_n + P_{0n} P_n^{(0)} + P_{1n} P_n^{(1)} \right) = \qquad (15)$$

$$\frac{2N_{\beta n} N_{nn}}{\varepsilon_n^2 N_\beta N_n} . \qquad (16)$$

Terms proportional to $P_n^{(2)}$ or $P_{2n}^{(0)}$, the $Q_{\beta(2n)}$ value for ^{29}Na being only 1.0 MeV, have been neglected.

The resultant neutron branches for ^{29}Ne are given in table 2. With the value of $\varepsilon_n = 0.19 \pm 0.01$ [3] as the efficiency of the neutron counter, the P_n value was determined from eq. (14), taking into account the uncertainties on all parameters. The error on P_n, which improves the literature value, is dominated by the uncertainties on $N_{\beta n}$, $P_n^{(0)}$ and ε_n. Even for data with infinitely good statistics, the error would be 3.5% rather than 5%. A mean residence time of 96 ± 8 μs resulted from the exponential fit of βn coincidences (fig. 3 right), in agreement with previous measurements [3].

In order to determine N_{nn}, time correlations between neutrons were analysed in two time regions consisting of: i) the beam related activity prior to 200 ms after proton impact, and ii) the background activity between 300 and 1200 ms. For the data taken at the HRS, which has the best statistics but contains no time information on β-particles, the first time region contains 20 n-n events (1000 μs correlation time used), subject to an expected background of 17 ± 2 events. Nearly all background events belong to the exponential component. Using a conservative estimate of 15 background events, an upper limit of $N_{nn} = 13.5$ (90% confidence level) follows [15], which in turn implies $P_{2n} < 2.2\%$ (90% confidence level) according to eq. (15). Similarly, using eq. (16), an upper limit of 2.9% for P_{2n} was determined from the GPS data. Thus, there is no indication of β-delayed two-neutron emission from ^{29}Ne, even though the Q-value for this process is 7.5 MeV.

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References

1. M.J.G. Borge *et al.*, Z. Phys. A **340**, 255 (1991).
2. M.J.G. Borge *et al.*, Nucl. Phys. A **613**, 199 (1997).
3. U.C. Bergmann *et al.*, Nucl. Phys. A **658**, 129 (1999).
4. U.C. Bergmann, K. Riisager, *Proceedings of RNB2000, the Fifth International Conference on Radioactive Nuclear Beams, Divonne, 2000*, to be published in Nucl. Phys. A; U.C. Bergmann, K. Riisager, work in preparation.
5. S. Baker, R.D. Cousins, Nucl. Instrum. Methods **221**, 437 (1984).
6. F. James, MINUIT, *Function Minimization and Error Analysis*, reference manual; CERN Program Library Long Writeup D506, 1994.
7. V.T. Koslowsky *et al.*, Nucl. Instrum. Methods A **401**, 289 (1997).
8. Y. Jading, K. Riisager, Nucl. Instrum. Methods A **372**, 289 (1996).
9. W.T. Eadie, D. Drijard, F. James, M. Roos, B. Sadoulet, *Statistical Methods in Experimental Physics* (North-Holland, 1971).
10. G. Audi *et al.*, Nucl. Phys. A **624**, 1 (1997).
11. D.E. Alburger, A.M. Nathan, Phys. Rev. C **17**, 280 (1978).
12. M. Notani *et al.*, in *Proceedings of ENAM98, Exotic Nuclei and Atomic Masses, Bellaire, 1998*, edited by B.M. Sherrill, D.J. Morrissey, C.N. Davids, AIP Conf. Proc. Vol. **455**, 359 (1998).
13. A.T. Reed *et al.*, Phys. Rev. C **60**, 024311 (1999).
14. O. Tengblad *et al.*, Z. Phys. A **342**, 303 (1992).
15. G.J. Feldman, R.D. Cousins, Phys. Rev. D **57**, 3873 (1998).